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Efficient surface and volume integration on implicitly defined domains using the moment-fitting equations

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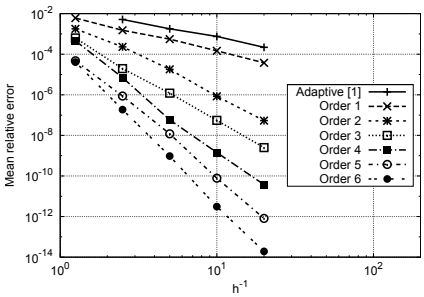
ABSTRACT

We present a new method for the numerical integration over domains that are defined by the zero iso-contour of a level set function. Such integrals commonly appear in methods dealing with non-trivial, sharp interfaces that are not aligned with the computational grid. Examples include the eXtended Finite Element Method (XFEM), the Finite Cell Method and variants of the Discontinuous Galerkin Method. They share the property that part of the burden of discretization is shifted to the numerical integration of generic functions over complicated domains for which conventional quadrature rules are hard if not impossible to obtain. As a consequence, the viability of the extension of these methods to higher approximation orders is limited by the affordable integration accuracy.

Classical methods like the regularization of singular or discontinuous integrands and the adaptive subdivision of intersected cells (e.g., [1]) are simple and easy to implement but are strongly limited in terms of the affordable accuracy. Recently, a method based on the *moment-fitting* equations has been introduced [2]. The authors use an optimization procedure to determine efficient quadrature rules for the computation of integrals in the context the XFEM. Unfortunately, this method is limited to a small class of problems due to the high cost of the optimization procedure. The presented approach is therefore based on a reformulation of the optimization procedure that strongly decreases the computational effort and still reaches very high convergence rates [3].



Grid size vs. error for the calculation of the arc-length of an ellipse on a Cartesian grid

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