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Pressure/velocity enrichtment for 3D two-phase incompressible flows

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ABSTRACT

In this talk 3D incompressible flow simulations of two-phase systems (liquid/liquid or liquid/gas) with finite elements on adaptive multilevel tetrahedral grids are considered, where a level set technique is applied for interface capturing. Compared to one-phase flows, the velocity and pressure field in two-phase flow problems are smooth in the interior of each phase Ω_1, Ω_2, but undergo certain singularities at the interface Γ between the phases. E. g., the pressure has a jump across Γ due to surface tension. The velocity is continuous across Γ, but different viscosities of the fluid phases induce a kink of the velocity field at the interface. The latter is especially the case for liquid-gas systems where the viscosity ratio is rather large.

The approximation of such functions with classical finite elements (conforming or non-conforming) leads to poor $\mathcal{O}(\sqrt{h})$ convergence, if the finite element grid is not aligned to the interface (which is typically the case for interface capturing methods like level set or VOF). The application of suitable extended finite element methods (XFEM) can provide optimal approximation properties. We present a Heaviside enrichment of the pressure space [1] yielding second order convergence of the $L_2(\Omega)$ pressure error [4] and consider a ridge enrichment [3] or Nitsche-XFEM for the velocity space leading to first order convergence of the $H^1(\Omega_1 \cup \Omega_2)$ velocity error. At the end of the talk, we present application examples of 3D droplet and falling film simulations obtained by our 3D two-phase flow solver DROPS [2, 5].

REFERENCES

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