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XFEM APPROXIMATION OF HETEROGENEOUS FLOWS

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ABSTRACT

We address a two-phase Stokes problem, namely the coupling of two fluids with different kinematic viscosities $\nu_1 \neq \nu_2$. More precisely, we aim to find velocity and pressure fields $\mathbf{u}_i$ and p_i respectively, such that

$$\begin{cases} -\nu_i \Delta \mathbf{u}_i + \nabla p_i = f_i, \quad \nabla \cdot \mathbf{u}_i = 0, & \text{in } \Omega_i, \\ \mathbf{u}_1 = \mathbf{u}_2, \quad p_1 \mathbf{n} - \nu_1 \nabla \mathbf{u}_1 \cdot \mathbf{n} = p_2 \mathbf{n} - \nu_2 \nabla \mathbf{u}_2 \cdot \mathbf{n} & \text{on } \Gamma, \end{cases} \tag{1}$$

complemented with suitable boundary conditions. We observe that the interface conditions allow the pressure and the velocity gradients to be discontinuous across the interface. XFEM is applied to accommodate the weak discontinuity of the velocity field across the interface and the jump in pressure. Numerical evidences, see [1] as well as the results reported in Figure 1, show that the discrete pressure approximation may be unstable in the neighborhood of the interface, even though the spatial approximation is based on *inf-sup* stable finite elements. Clearly, XFEM enrichment locally violates the satisfaction of the stability condition for mixed problems. For this reason, resorting to some pressure stabilization technique in the region of elements cut by the unfitted interface is recommended. The *ghost penalty* stabilization, see [1] and references therein, could fit to this case. However, it substantially affect the sparsity pattern of the algebraic system of equations, with possible drawbacks at the level of numerical solvers. In alternative, we consider the application of stabilized equal order pressure / velocity XFEM discretizations and we analyze their approximation properties. On one side, this strategy increases the flexibility on the choice of velocity and pressure approximation spaces. On the other side, some pressure stabilization operators, such as local pressure projection methods or the Brezzi-Pitkaranta scheme, seem to be effective to cure the additional source of instability arising from the XFEM approximation. These operators could be applied *locally*, namely only in proximity of the interface. Besides a rigorous stability and error analysis, numerical results on benchmark cases will be discussed, in order to thoroughly compare the performance of the aforementioned methods.

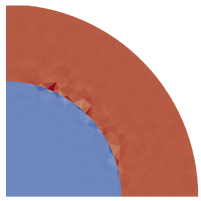

Figure 1: Approximation of (1) with enriched $P1$-bubble / $P1$ elements.

REFERENCES

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