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A. Gravouil, Y. Renard and A. Combescure

A PRIORI ESTIMATES FOR STRONG OR WEAK CURVED DISCONTINUITY PROBLEMS

Guilhem Ferté[1,2], Patrick Massin[1], Nicolas Moës[2]
[1] LaMSID, UMR EDF - CNRS - CEA 8193,
EDF R&D, 1 av. du général de Gaulle, 92141 Clamart, France
{guilhem.ferte, patrick.massin}@edf.fr

[2] GeM, UMR Centrale Nantes - Université de Nantes - CNRS 6183
1 rue de la Noë, BP 92101, 44321 Nantes, France
nicolas.moes@ec-nantes.fr

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ABSTRACT

The aim of this talk is to provide *a priori* error estimates in the X-FEM method for curved interface problems with strong or weak discontinuities, the latter being handled with Lagrange multipliers. The interface geometry is approximated by means of higher-order interpolated level-sets and curved isoparametric subcells. The accuracy of this description is quantified using the appropriate criterion of [1]. Its influence over the convergence rate of the problem has been numerically investigated by numerous authors (see e.g. [2]), but there exists to our knowledge no theoretical investigation of the effect with X-FEM. Extending to X-FEM the work of [3] and [4], the convergence rate for strong discontinuities is deduced as a function of the above criterion and the quadrature scheme in the subcells.

As for weak discontinuity problems, the Lagrange DOF are defined on the vertices of the parent mesh and the discrete space is deduced imposing relations between some of these nodes (see e.g. [5]). We define a P1 multiplier space intended for use together with P2 displacements, with less relations than its P1/P1 counterpart in [5]. Analytical evidence that it passes the inf-sup condition is presented. *A priori* estimates inspired from [6] are derived for X-FEM weak discontinuity problems as functions of the geometry description, the inf-sup constant and the interpolation order of the multiplier. The predictions are in good accordance with the numerical results.

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