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Nitsche XFEM for multi-physics coupling problems

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ABSTRACT

In this communication, we will discuss XFEM methods using Nitsche type coupling conditions. Nitsche's method offers a flexible approach to the design of XFEM methods that also is amenable to analysis. Here, we are interested in the application of Nitsche's method to systems with different physical properties that are coupled across an interface that cuts through the elements. Nitsche's method has several advantages over alternative methods that can be used to enforce weak interface coupling conditions, such as Lagrange multiplier or penalty methods. In contrast to classical Lagrange multiplier methods no other unknown is introduced in the Nitsche method and no discrete inf-sup condition needs to be satisfied. And conversely to standard penalty parameter techniques, the resulting Nitsche method is consistent [1]. However, the interface boundary may cut the computational mesh in such a way that some of the resulting subelements belonging to different materials become very small and the linear system arising from Nitsche's discrete problem may be ill-posed [2]. To overcome this difficulty, we suggest the addition of ghost-penalty terms to the variational formulation over the band of elements that are cut by the interface [2,3]. This method has been applied by Burman and Zunino [3] for elliptic problems with a large jump in the coefficients across the interface. In this contribution, we will first extend and apply this technique to the computation of material properties in composite structures. Then, we will discuss how the method can be extended to the case of the three field Stokes' problem, both in the case of multi-physics coupling across the interface and the case of the fictitious domain method. The implementation of the method is realised in the finite element software package FEniCS [4].

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