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SPECTRAL/HP-ADAPTIVE XFEM WITH APPLICATIONS TO TWO-PHASE FLOW PROBLEMS

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ABSTRACT

Two-phase flow problems involving the motion of curved interfaces require the characterisation of arbitrary curved strong and weak discontinuities to accurately resolve the pressure and velocity fields.

The Extended Finite Element Method (XFEM), together with interface capturing via level sets, allows such problems to be solved using a fixed background mesh. This avoids the presence of distorted elements and the need for frequent remeshing associated with boundary-fitted methods where the mesh is updated in response to interface motion. Additionally, if a structured mesh is chosen then existing techniques for structured mesh adaptivity [1] may be used to improve the approximation.

Here we present a spectral/hp-adaptive XFEM with optimal or close-to-optimal convergence for curved strong and weak discontinuities, based on [2–4]. Convergence rates are presented for truly high order ($p > 3$) approximation. We apply the method to fluid interface problems with surface tension involving droplet merging and splitting and interface overturning where boundary-fitted methods may struggle.

We also investigate the use of space-time methods [5] in order to overcome problems associated with the coupling of the level set function to the velocity and pressure fields. The motion of the interface causes the enriched basis functions to change between timesteps, an effect that must be included in the solution of the discrete nonlinear system. In addition, changes in the number of degrees of freedom and linear dependence among enriched basis functions can also occur as the interface crosses between elements. The use of space-time methods alleviates these problems, but at the expense of working in a higher dimensional space.

REFERENCES

[1] G. Karniadakis and S. Sherwin. *Spectral/hp Element Methods for Computational Fluid Dynamics*, 2nd Edition, Oxford University Press, 2005.

[2] K.W. Cheng and T.-P. Fries. Higher-order XFEM for curved strong and weak discontinuities, *Int. J. Numer. Meth. Eng.*, **82**:654–590, 2010.

[3] A. Legay, H.W. Wang and T. Belytschko. Strong and weak arbitrary discontinuities in spectral finite elements, *Int. J. Numer. Meth. Eng.*, **64**:991–1008, 2005.

[4] N. Moës, M. Cloirec, P. Cartraud and J. F. Remacle. A computational approach to handle complex microstructure geometries, *Comput. Method. Appl. M.*, **192**:3163–3177, 2003.

[5] J. Chessa and T. Belytschko. Arbitrary discontinuities in space-time finite elements by level-sets and X-FEM, *Int. J. Numer. Meth. Eng.*, **61**:2595–2614, 2004.