International Conference on Extended Finite Element Methods - XFEM 2013
September 11 – 13, 2013, Lyon, France
A. Gravouil, Y. Renard and A. Combescure

Performance of ILU preconditioning combined with wavelet transformation for numerical solution by PUMFEM of 3D elastic wave problems

M. S. Mahmood, O. Laghrouche, A. El-Kacimi
Institute for Infrastructure and Environment, Heriot-Watt University, Edinburgh EH14 4AS, UK

m.mahmood@hw.ac.uk, o.laghrouche@hw.ac.uk, a.elkacimi@hw.ac.uk

J. Trevelyan
School of Engineering and Computing Sciences, Durham University, Durham DH1 3LE, UK

jon.trevelyan@durham.ac.uk

Key Words: *PUFEM, finite element method, plane waves, elastic waves, 3D, error estimate.*

ABSTRACT

Elastic wave propagation modelling arises in many engineering applications, including traffic vibrations from roads and railways, seismic induced vibrations and foundation construction, etc. The numerical modelling of these problems, in frequency domain by the conventional Finite Element Method (FEM), requires finite element grids sufficiently fine in comparison with the wavelengths, to get accurate results. When typically, the piecewise linear finite element is implemented, around ten nodal points per lower wavelength are needed, to ensure adequate resolution of the wave pattern. However, in the case of high frequency (small wavelength) and/or large domain of interest, the finite element mesh requires a large number of elements, and consequently the procedure becomes computationally expensive and impractical.

This work is devoted to develop finite elements, for three dimensional elastic wave problems, capable of containing many wavelengths per nodal spacing. This will be achieved by applying the plane wave basis decomposition to the 3D elastic wave equation. These elements will allow us to relax the traditional requirement of around ten nodal points per wavelength and therefore solve elastic wave problems without refining the mesh of the computational domain at each frequency. The accuracy and effectiveness of the proposed technique will be determined by comparing solutions for selected problems with available analytical solutions and/or to high resolution numerical solutions using conventional finite elements [1].

In spite of the reduced number of degrees of freedom per wavelength, the three dimensional wave based FE approach leads to extremely large matrices with large bandwidth. Moreover, these are highly indefinite and extremely ill conditioned. Efficient preconditioners are derived using ILU preconditioners in krylov subspace iterative solution in conjunction with the discrete wavelet transformation [2].

REFERENCES

[1] M. S. Mahmood, O. Laghrouche, A. El Kacimi and J. Trevelyan. Elastic wave numerical modelling in 3D by the partition of unity finite element method, *Int. J. Numer. Meth. Engng.*, 2013 (submitted).

[2] A. El Kacimi, and O. Laghrouche. Wavelet based ILU preconditioners for the numerical solution by PUFEM of high frequency elastic wave scattering. *J. Comp. Phys.*, Vol. **230**, 3119–3134, 2011.