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Error analysis for numerical solution by PUMFEM of 3D elastic wave problems

M. S. Mahmood, O. Laghrouche
Institute for Infrastructure and Environment, Heriot-Watt University Edinburgh EH14 4AS, UK
m.mahmood@hw.ac.uk, o.laghrouche@hw.ac.uk

J. Trevelyan
School of Engineering and Computing Sciences, Durham University, Durham DH1 3LE, UK
jon.trevelyan@durham.ac.uk

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ABSTRACT

This work is devoted to analyze a partition of unity finite element methods (PUFEM) for three dimensional elastic wave problems. This approach provide finite elements capable of containing many wavelengths per nodal spacing and have been very successful in reducing the computing effort associated to the high frequency, see and the references therein. PUFEM has been shown capable of containing many wavelengths per nodal spacing. This will be achieved by applying the plane wave basis decomposition to the 3D elastic wave equation.

Let Ω be a bounded domain with Lipschitz continuous boundary in $\mathbf{R}^d$, $d = 3$, occupied by an elastic medium. Let us denote by $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ the cartesian vector system, and $\mathbf{x} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3$ a generic point in $\mathbf{R}^d$. The three dimensional propagation model of time harmonic elastic waves in homogenous isotropic medium is governed by the Navier equation, see [1]:

$$-\rho\omega^2\mathbf{U} - \nabla \cdot \sigma(\mathbf{U}) = \rho\mathbf{F}, \tag{1}$$

where ρ is the density of the medium, ω is the circular frequency and $\mathbf{F}$ is a body force. The stress tensor σ, evaluated at a displacement $\mathbf{U} = U_1\mathbf{e}_1 + U_2\mathbf{e}_2 + U_3\mathbf{e}_3$, is given by the classical Hooke's law

$$\sigma(\mathbf{U}) = \lambda\nabla \cdot \mathbf{U}\,\mathbf{I} + \mu(\nabla\mathbf{U} + \nabla\mathbf{U}^\top), \tag{2}$$

where $\mathbf{I}$ is the identity matrix in $\mathbf{R}^d \times \mathbf{R}^d$, λ and μ are the Lamé parameters of the elastic material, assumed constant, and $\nabla\mathbf{U} = (\nabla U_1, \nabla U_2, \nabla U_3)^\top$.

We derive the error estimates and study the convergence analysis of our PUFEM for the problem above. This error analysis is motivated mainly by the work, see [2]. The error analysis is based on the regularity assumptions to the weak solution. Apiori estimates are carried out based on several previous works by [2] and others. This error is also depends on the approximation properties of the plane wave basis functions for linear elasticity where uniform number of 3D basis functions are used.

REFERENCES

[1] M. S. Mahmood, O. Laghrouche, A. El Kacimi and J. Trevelyan. Elastic wave numerical modelling in 3D by the partition of unity finite element method, *Int. J. Numer. Meth. Engng.*, 2013 (submitted).

[2] T. Luostari, T. Huttunen and P. Monk. Error estimates for the Ultra Weak Variational Formulation in linear elasticity. *ESAIM: Math. Mod. and Numer. Anal.*, Vol. **47**, 183–211, 2012.